Analysis of the Spatial Interpolation Error associated with Maps of Median Annual Climate Variables

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Contents

Disclaimer

1. Description of the spatial interpolation methodology

Mid-latitude Southern Hemisphere westerlies and associated weather systems passing over New Zealand interact with the country's complex terrain to produce substantial spatial variations in the climate (Coulter, 1975; Salinger et al., 2004). To produce maps showing the full spatial variation of New Zealand's climate, information is needed with a spatial resolution of 500m or better. This requires spatial interpolation of meteorological data from irregularly spaced climate stations onto high resolution regular grids, through a procedure which can reflect the main causes of inter-station variability (Wratt *et al*., 2006). Hutchinson (1991, 1995) describes the method of thin plate (or Laplacian) smoothing spline interpolation, and its application to the interpolation of climate data. This method is summarised in the following paragraphs.

Many standard interpolation packages use bivariate interpolation, or interpolation based on two variables (usually x and y location parameters, e.g. latitude and longitude). For example, inverse distance weighting (IDW) is an interpolation method that estimates values between locations using a weighted average of the data from those locations, where the weights are the inverse of the distance from the data sources to the interpolation point. Bivariate interpolation therefore requires detailed information about the location of the data (e.g. climate stations). However, these methods do not incorporate any dependencies on additional geographic variables, such as elevation and distance from the sea, which are often highly significant. Thin plate smoothing splines provide a robust way of incorporating any number of additional variables that may significantly improve the accuracy of the interpolation, in addition to location.

Splines work by fitting a surface to the data with some error allowed at each data point, so the surface can be smoother than if the data were fitted exactly. A single parameter controls the smoothing and is normally chosen to minimise the mean square error between the actual value at the stations and their values predicted by all the other stations. Each station is omitted in turn from the estimation of the fitted surface and the mean error is found. This is repeated for a range of values of the smoothing parameter, and the value that minimises the mean error is taken to give the optimum smoothing. This is called the method of generalised cross validation (GCV).

Most meteorological variables, such as air temperature, rainfall and evaporation, are affected by altitude. Thus it makes sense to interpolate these parameters using a spline model with two position variables (latitude and longitude or easting and northing) and a sub-model which accounts for the dependence on elevation. The broad spatial pattern is determined by the two position variables, while the inclusion of elevation modifies the broad pattern to give more precise representations of the higher resolution variability. An advantage of the spline formulation is that the coefficient of the linear sub-model (e.g. the temperature rate of change with elevation, or the lapse rate) is determined automatically from the data (Hutchinson, 1991), and therefore does not need to be specified *a priori*. Additional geographic variables such as distance away from the coast or proximity to the main divide, or meteorological variables such as percent cloud cover (for mapping solar radiation) can also be included in the spline model.

Maps of long-term statistics (e.g. the median and the 20th and 80th percentiles), based on the climatological period 1971–2000, have been produced for New Zealand for several climate parameters using the spline model ANUSPLIN (Version 4.3: Hutchinson, 2007). A trivariate (three independent variables) thin plate smoothing spline was used, as this model consistently resulted in lower interpolation errors over the whole country compared with bivariate and bivariate plus a dependent variable models. This result is similar to an analysis of spline model options for interpolating climate variables for Australia (Hutchinson, 1989).

For the New Zealand maps, the third variable in the interpolation is dependent upon the climate parameter being mapped (Table 1). Elevation above sea level was used for temperature-related parameters such as the minimum, mean, and maximum air temperature; soil temperature at 10cm depth, growing degree days base 5 and 10°C, cold season chilling total, potential evapotranspiration, and relative humidity. Rainfall total, number of days of rain, and days of soil moisture deficit were interpolated using an expert-guided map of the 1951–80 mean annual rainfall as the third independent variable (Tait *et al*., 2006). The average percent of days with cloud cover (derived from satellite observations) was used to interpolate total sunshine hours and daily solar irradiance, and the average annual daily temperature range (derived from spatial interpolation using elevation as the third variable in the spline) was used to interpolate mean daily wind speed.

Hutchinson (1989) used a trivariate thin plate smoothing spline to interpolate several meteorological variables across Australia. The number of data points varied with meteorological parameter; 150 points (i.e. climate stations) for solar radiation, 300 for pan evaporation, 900 for maximum and minimum air temperature, and 10,000 points for precipitation. The approximate standard errors calculated within the spline program for this Australian study were: Solar radiation 3%; pan evaporation 5%; minimum and maximum temperature 0.2–0.5°C; rainfall 10–15%, respectively. As is shown in section 3 below, the errors for mapping climate variables for New Zealand are in a similar range to those for Australia.

2. Description of the interpolation model prediction error

The ANUSPLIN program used for this analysis produces as part of the suite of possible outputs an estimate of the model prediction standard error. This error statistic can be calculated at all of the data sites used for the interpolation. The model prediction standard error is a combination of the variance of the data values and the model standard error estimate of the interpolated values.

Confidence intervals of the calculated spline values are estimated by multiplying the prediction standard error by 1.96, the 95 percent two-sided confidence interval of the standard normal distribution.

3. Maps and error statistics

Table 1 shows some standard statistical outputs from ANUSPLIN for each climate parameter. The table lists the third independent variable used in the trivariate spline model (along with the two positional variables, easting and northing), the signal to noise ratio resulting from the GCV analysis, the root GCV, and the surface root mean square error.

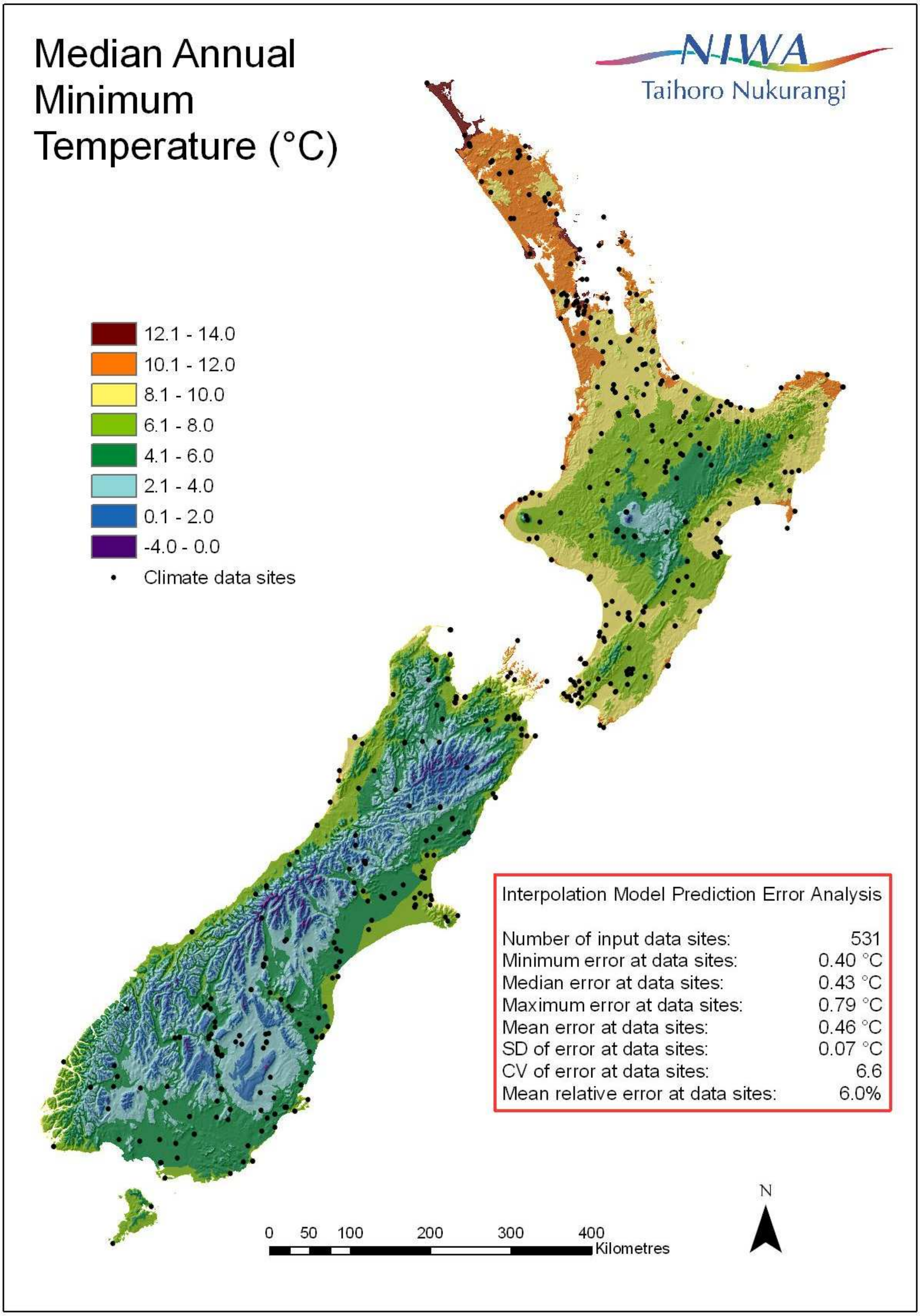
The climate maps presented in this section are the median annual statistic (e.g. the median annual rainfall total), based on the period 1971–2000, for 15 climate parameters. The parameters are:

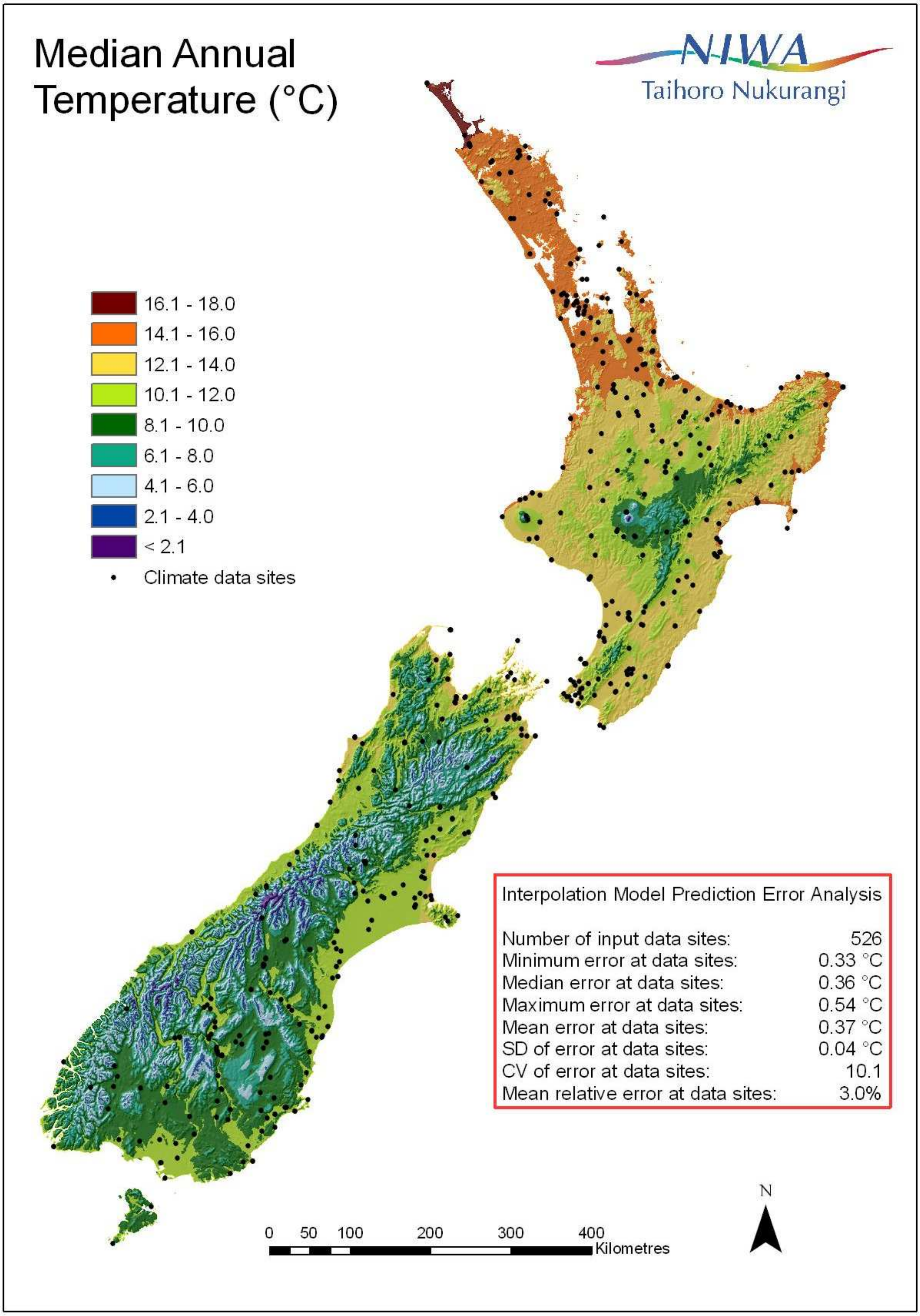
1. Median annual daily minimum air temperature
2. Median annual daily mean air temperature
3. Median annual daily maximum air temperature
4. Median annual daily soil temperature at 10cm
5. Median annual growing degree days base 5°C
6. Median annual growing degree days base 10°C
7. Median annual cold season chilling total
8. Median annual potential evapotranspiration total
9. Median annual 9am relative humidity
10. Median annual rainfall total
11. Median annual number of days of rain ≥ 1mm
12. Median annual number of days of soil moisture deficit
13. Median annual sunshine hours total
14. Median annual daily mean solar irradiance
15. Median annual daily mean wind speed

Each map includes the location of the climate stations from which data were used in the interpolation of the parameter. The error analysis box on each map describes the number of data sites, the minimum, median, maximum, mean, standard deviation, coefficient of variation, and mean relative prediction standard error at the data sites.

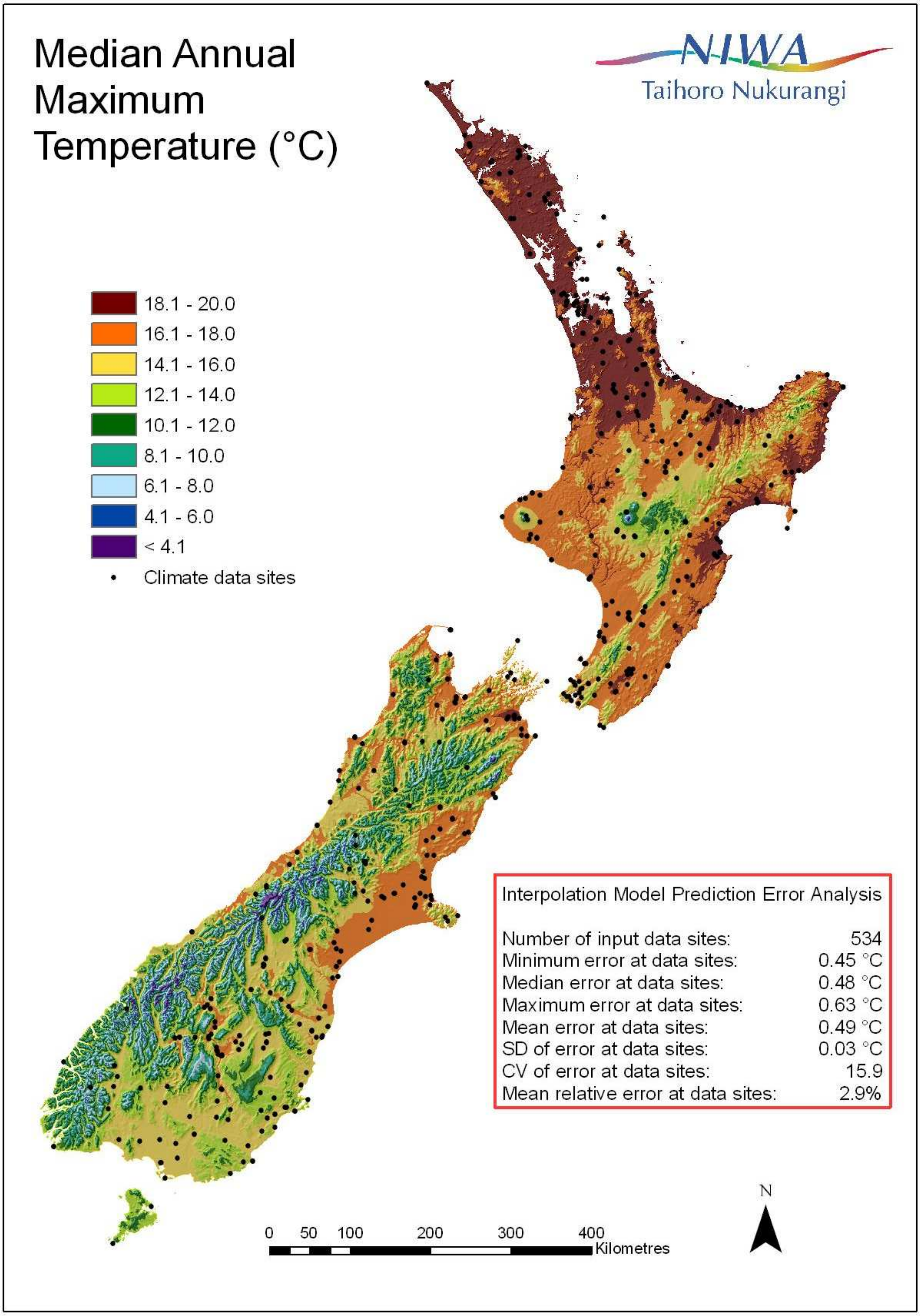
Table 1: Summary statistics from trivariate spline models for the interpolation of several climate parameters for New Zealand.

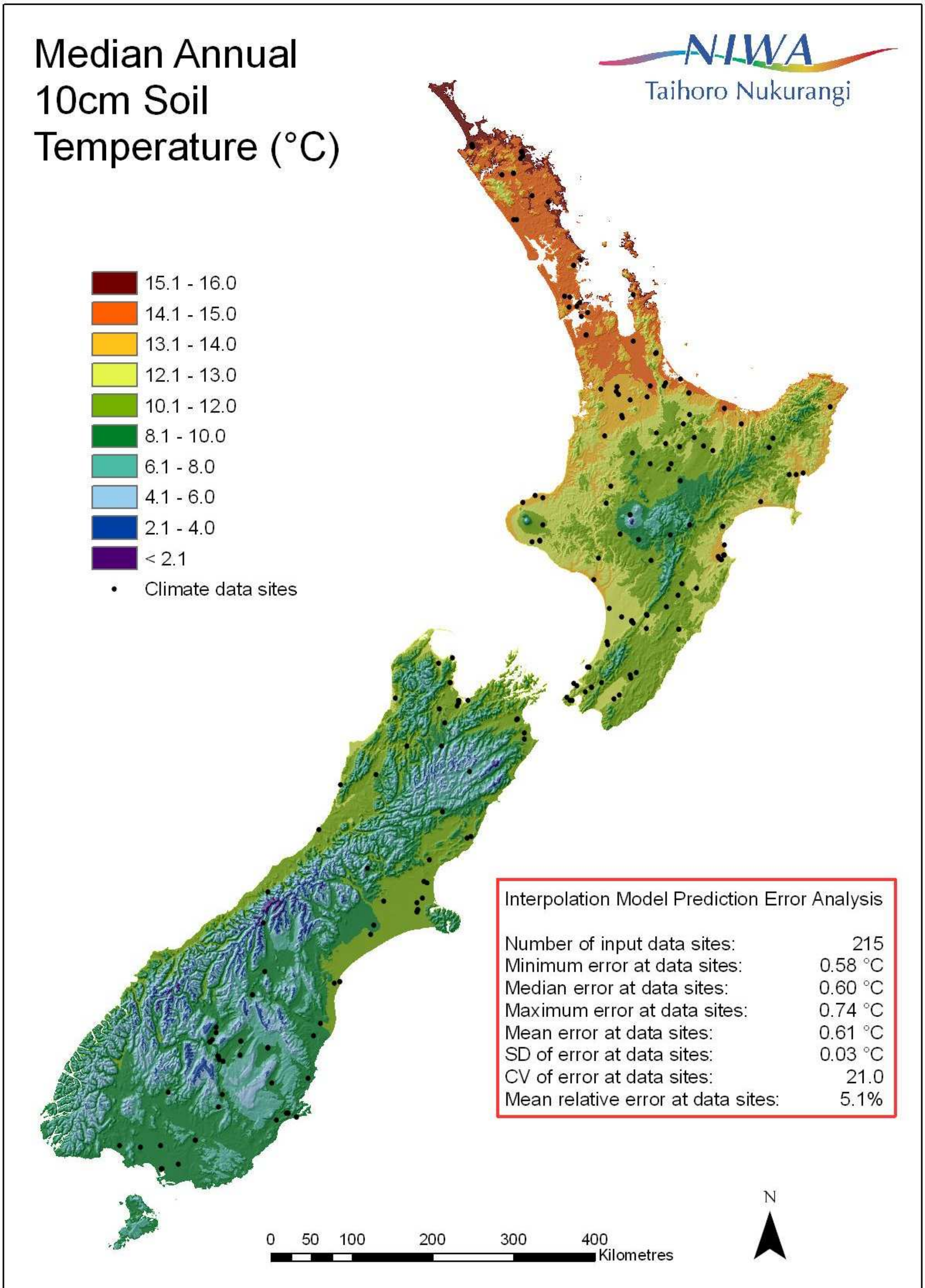
Climate parameter	**Third independent variable**	**Signal to Noise ratio**	**Root GCV**	**Root Mean Square Error**
Median annual daily minimum air temperature	Elevation above sea level (m)	2.77 : 1	0.73 °C	0.32 °C
Median annual daily mean air temperature	Elevation above sea level (m)	1.46 : 1	0.50 °C	0.25 °C
Median annual daily maximum air temperature	Elevation above sea level (m)	1 : 1.05	0.61 °C	0.30 °C
Median annual daily soil temperature at 10cm	Elevation above sea level (m)	1 : 1.67	0.70 °C	0.34 °C
Median annual growing degree days base 5°C	Elevation above sea level (m)	1.45 : 1	178 gdds	87.4 gdds
Median annual growing degree days base 10°C	Elevation above sea level (m)	1.34 : 1	138 gdds	68.3 gdds
Median annual cold season chilling total	Elevation above sea level (m)	1 : 1.16	194 hours	96.5 hours
Median annual potential evapotranspiration total	Elevation above sea level (m)	1 : 1.01	36.8 mm	18.4 mm
Median annual 9am relative humidity	Elevation above sea level (m)	1.20 : 1	3.29 %	1.64 %
Median annual rainfall total	1951-80 mean annual rainfall (mm)	1.85 : 1	215 mm	103 mm
Median annual number of days of rain ≥ 1mm	1951-80 mean annual rainfall (mm)	1 : 1.01	13.9 days	6.9 days
Median annual number of days of soil moisture deficit	1951-80 mean annual rainfall (mm)	1 : 1.33	15.8 days	7.82 days
Median annual sunshine hours total	Mean percent of cloud cover (%)	2.72 : 1	161 hours	71.5 hours
Median annual daily mean solar irradiance	Mean percent of cloud cover (%)	1.85 : 1	0.79 $MJ/m^2/day$	0.38 $MJ/m^2/day$
Median annual daily mean wind speed	Mean annual daily temperature range (°C)	2.98 : 1	1.16 m/s	0.51 m/s

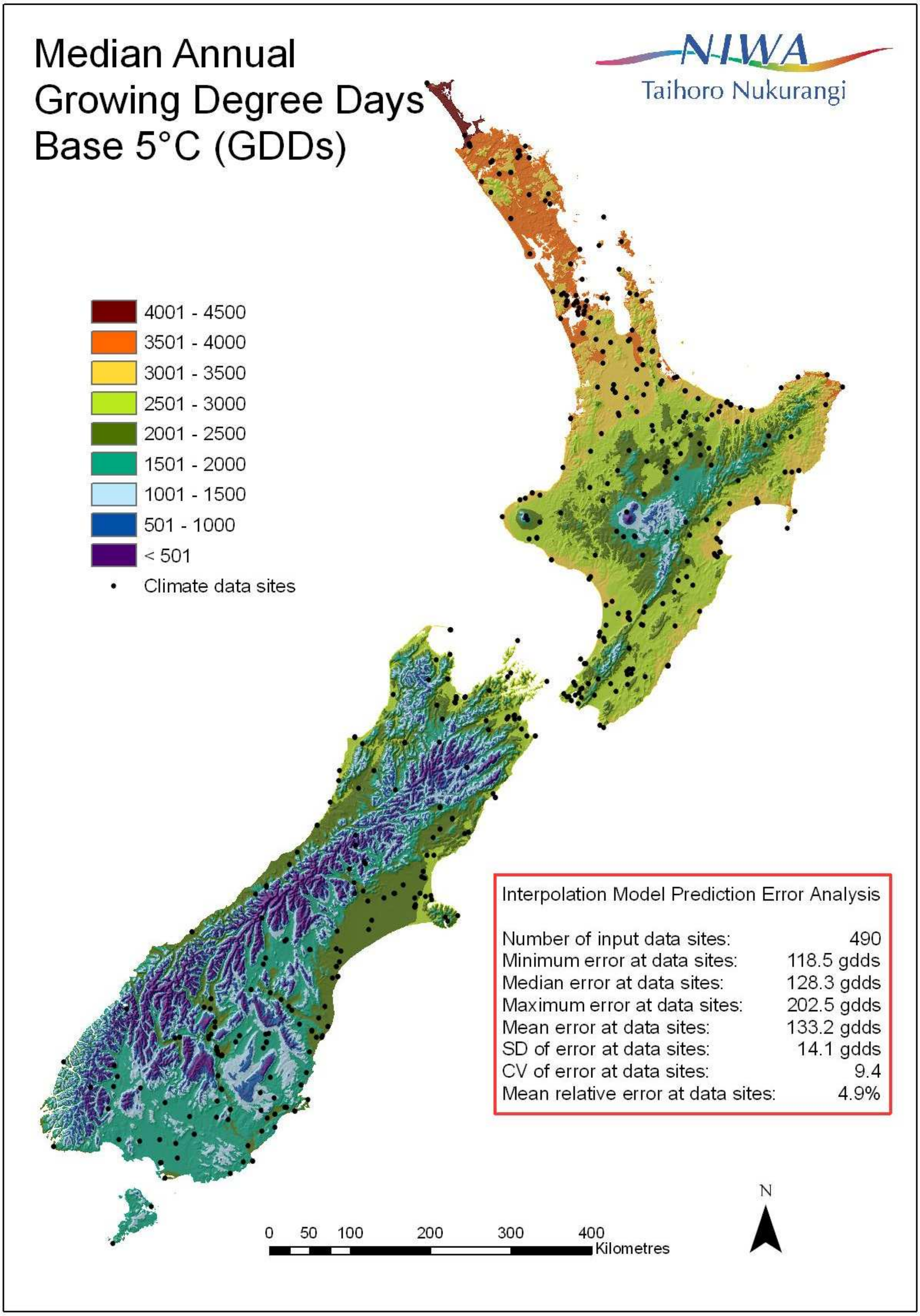

Median Annual Minimum Temperature (°C)
NIWA
Taihoro Nukurangi
12.1 - 14.0
10.1 - 12.0
8.1 - 10.0
6.1 - 8.0
4.1 - 6.0
2.1 - 4.0
0.1 - 2.0
-4.0 - 0.0
Climate data sites
Interpolation Model Prediction Error Analysis
Number of input data sites: 531
Minimum error at data sites: 0.40 °C
Median error at data sites: 0.43 °C
Maximum error at data sites: 0.79 °C
Mean error at data sites: 0.46 °C
SD of error at data sites: 0.07 °C
CV of error at data sites: 6.6
Mean relative error at data sites: 6.0%
0 50 100 200 300 400
Kilometres
N

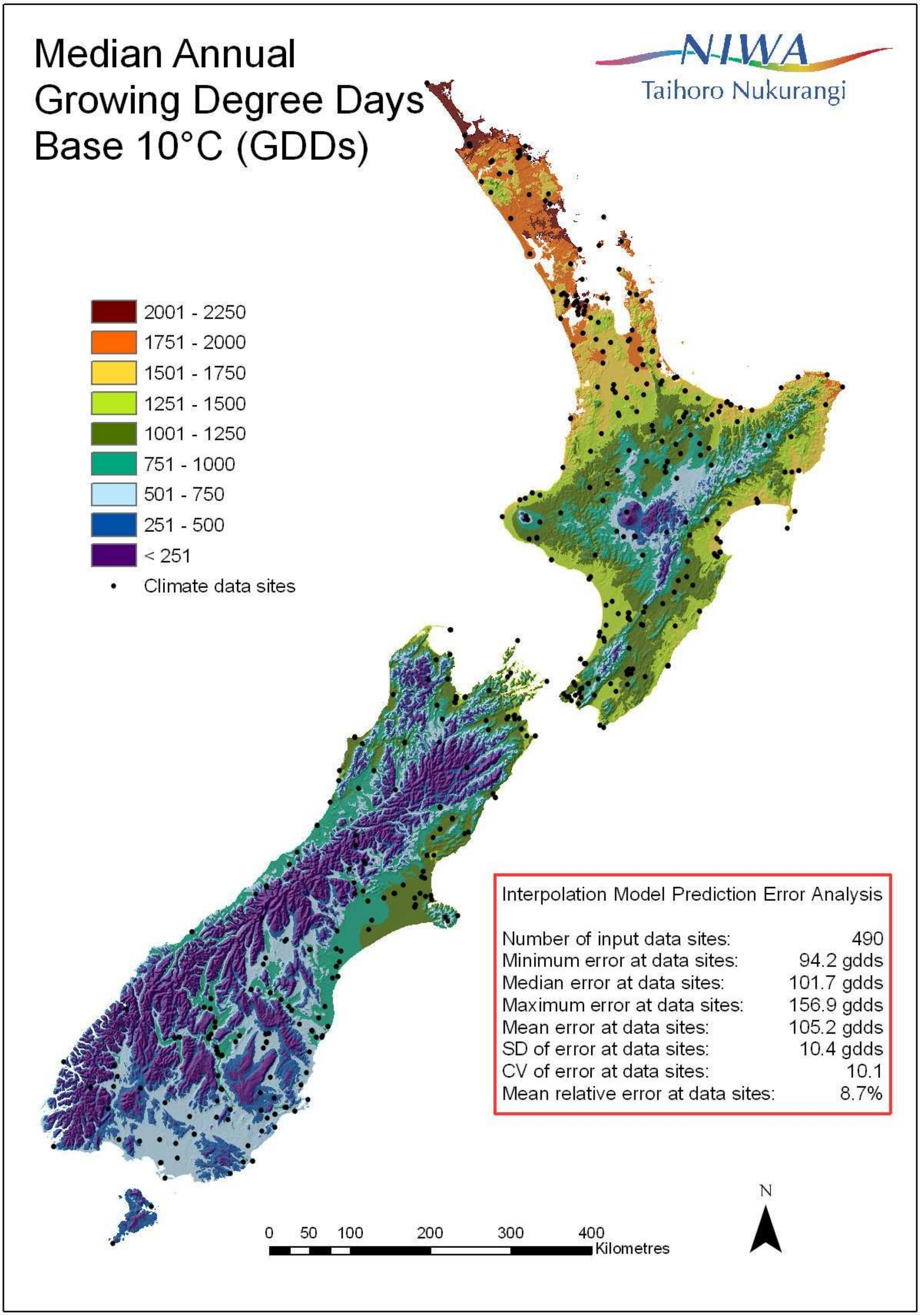


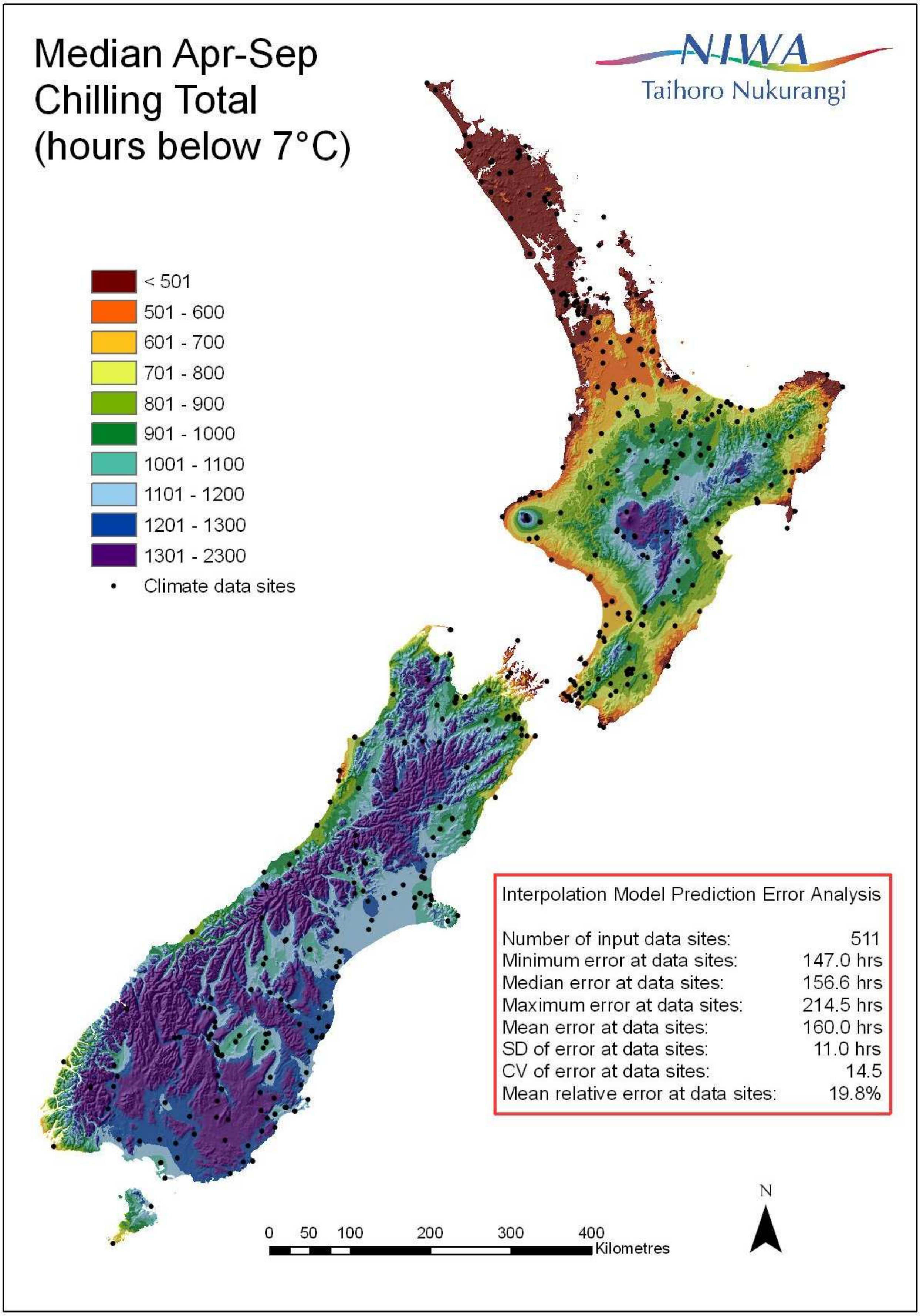
Median Annual Temperature (°C)
NIWA
Taihoro Nukurangi
16.1 - 18.0
14.1 - 16.0
12.1 - 14.0
10.1 - 12.0
8.1 - 10.0
6.1 - 8.0
4.1 - 6.0
2.1 - 4.0
< 2.1
Climate data sites
Interpolation Model Prediction Error Analysis
Number of input data sites: 526
Minimum error at data sites: 0.33 °C
Median error at data sites: 0.36 °C
Maximum error at data sites: 0.54 °C
Mean error at data sites: 0.37 °C
SD of error at data sites: 0.04 °C
CV of error at data sites: 10.1
Mean relative error at data sites: 3.0%
0 50 100 200 300 400
Kilometres
N

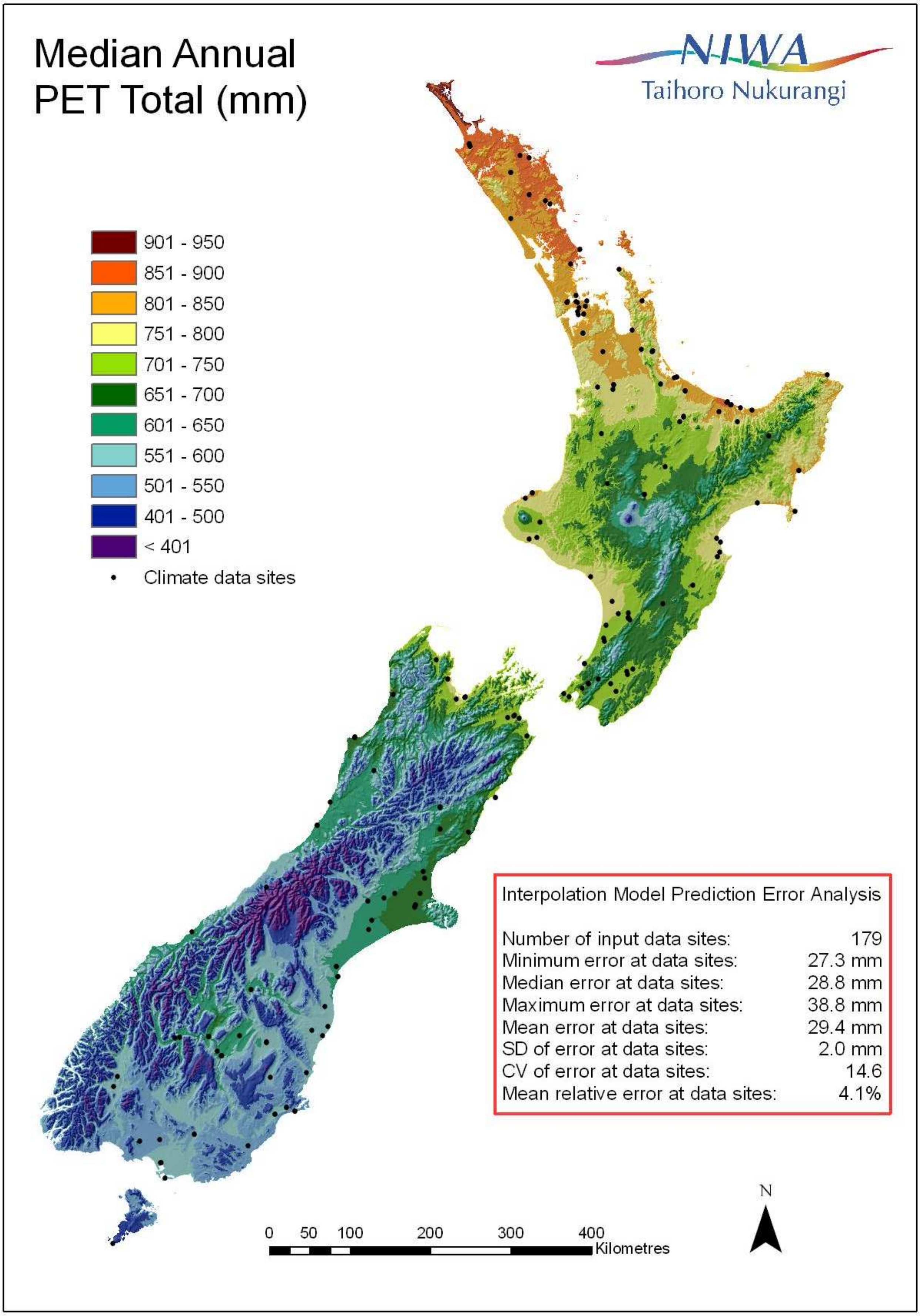

Median Annual Maximum Temperature (°C)
NIWA
Taihoro Nukurangi
18.1 - 20.0
16.1 - 18.0
14.1 - 16.0
12.1 - 14.0
10.1 - 12.0
8.1 - 10.0
6.1 - 8.0
4.1 - 6.0
< 4.1
Climate data sites
Interpolation Model Prediction Error Analysis
Number of input data sites: 534
Minimum error at data sites: 0.45 °C
Median error at data sites: 0.48 °C
Maximum error at data sites: 0.63 °C
Mean error at data sites: 0.49 °C
SD of error at data sites: 0.03 °C
CV of error at data sites: 15.9
Mean relative error at data sites: 2.9%
0 50 100 200 300 400
Kilometres
N


Median Annual
10cm Soil
Temperature (°C)
NIWA
Taihoro Nukurangi
15.1 - 16.0
14.1 - 15.0
13.1 - 14.0
12.1 - 13.0
10.1 - 12.0
8.1 - 10.0
6.1 - 8.0
4.1 - 6.0
2.1 - 4.0
< 2.1
Climate data sites
Interpolation Model Prediction Error Analysis
Number of input data sites: 215
Minimum error at data sites: 0.58 °C
Median error at data sites: 0.60 °C
Maximum error at data sites: 0.74 °C
Mean error at data sites: 0.61 °C
SD of error at data sites: 0.03 °C
CV of error at data sites: 21.0
Mean relative error at data sites: 5.1%
0 50 100 200 300 400
Kilometres
N

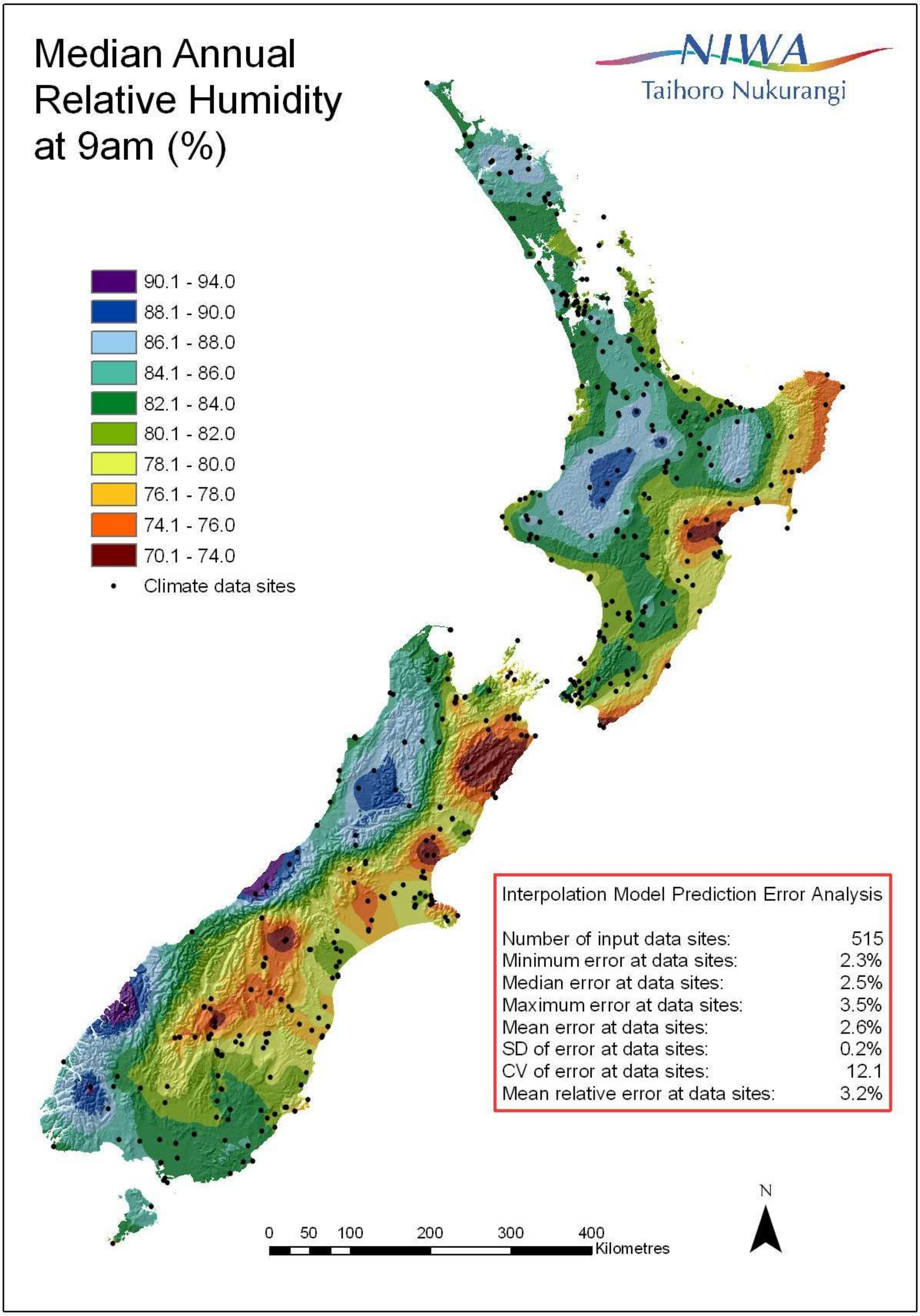

Median Annual
Growing Degree Days
Base 5°C (GDDs)
NIWA
Taihoro Nukurangi
4001 - 4500
3501 - 4000
3001 - 3500
2501 - 3000
2001 - 2500
1501 - 2000
1001 - 1500
501 - 1000
< 501
Climate data sites
Interpolation Model Prediction Error Analysis
Number of input data sites: 490
Minimum error at data sites: 118.5 gdds
Median error at data sites: 128.3 gdds
Maximum error at data sites: 202.5 gdds
Mean error at data sites: 133.2 gdds
SD of error at data sites: 14.1 gdds
CV of error at data sites: 9.4
Mean relative error at data sites: 4.9%
N
0 50 100 200 300 400
Kilometres

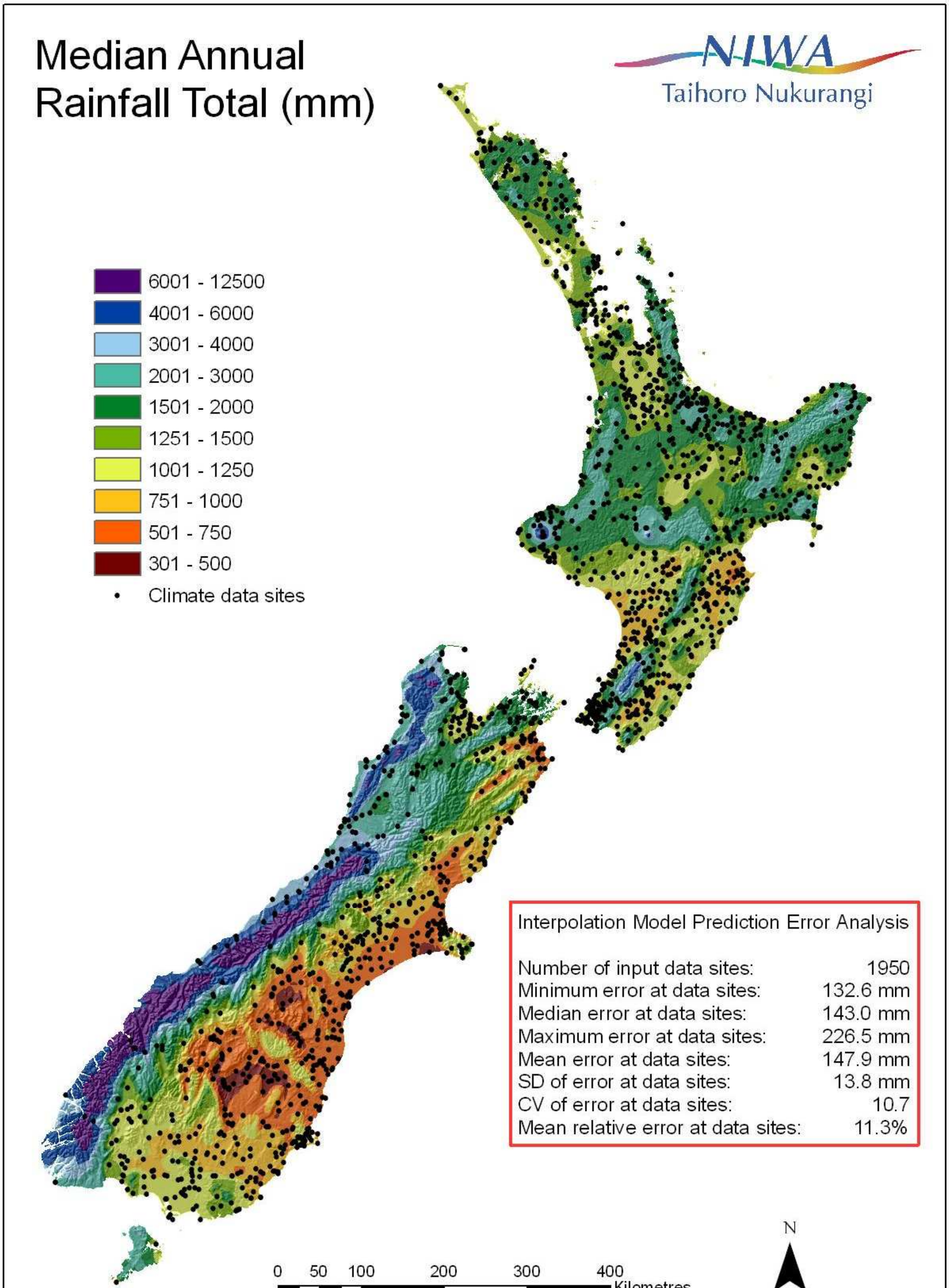

Median Annual
Growing Degree Days
Base 10°C (GDDs)
NIWA
Taihoro Nukurangi
2001 - 2250
1751 - 2000
1501 - 1750
1251 - 1500
1001 - 1250
751 - 1000
501 - 750
251 - 500
< 251
Climate data sites
Interpolation Model Prediction Error Analysis
Number of input data sites: 490
Minimum error at data sites: 94.2 gdds
Median error at data sites: 101.7 gdds
Maximum error at data sites: 156.9 gdds
Mean error at data sites: 105.2 gdds
SD of error at data sites: 10.4 gdds
CV of error at data sites: 10.1
Mean relative error at data sites: 8.7%
0 50 100 200 300 400
Kilometres
N

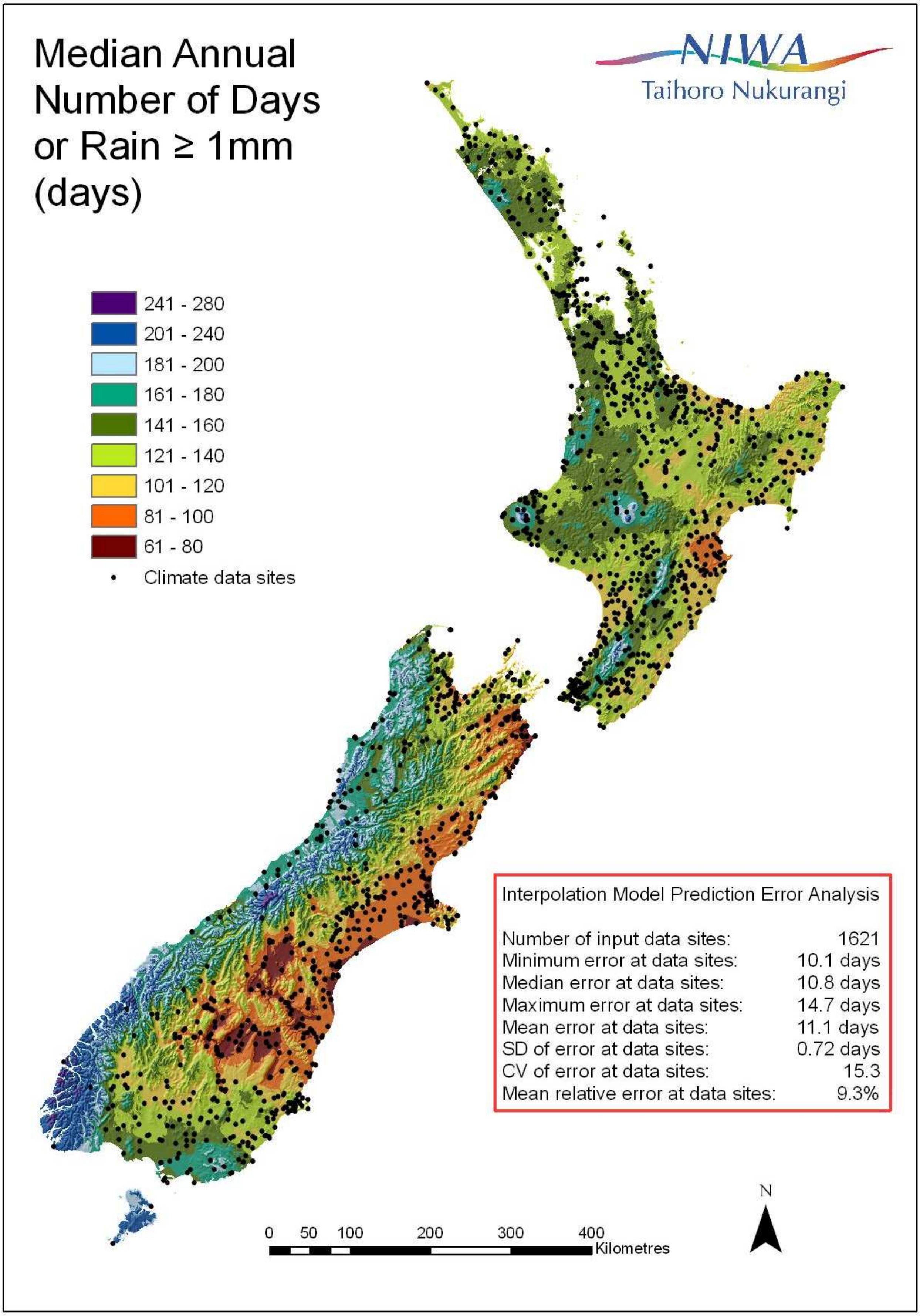

Median Apr-Sep Chilling Total (hours below 7°C)
NIWA
Taihoro Nukurangi
< 501
501 - 600
601 - 700
701 - 800
801 - 900
901 - 1000
1001 - 1100
1101 - 1200
1201 - 1300
1301 - 2300
Climate data sites
Interpolation Model Prediction Error Analysis
Number of input data sites: 511
Minimum error at data sites: 147.0 hrs
Median error at data sites: 156.6 hrs
Maximum error at data sites: 214.5 hrs
Mean error at data sites: 160.0 hrs
SD of error at data sites: 11.0 hrs
CV of error at data sites: 14.5
Mean relative error at data sites: 19.8%
0 50 100 200 300 400
Kilometres
N

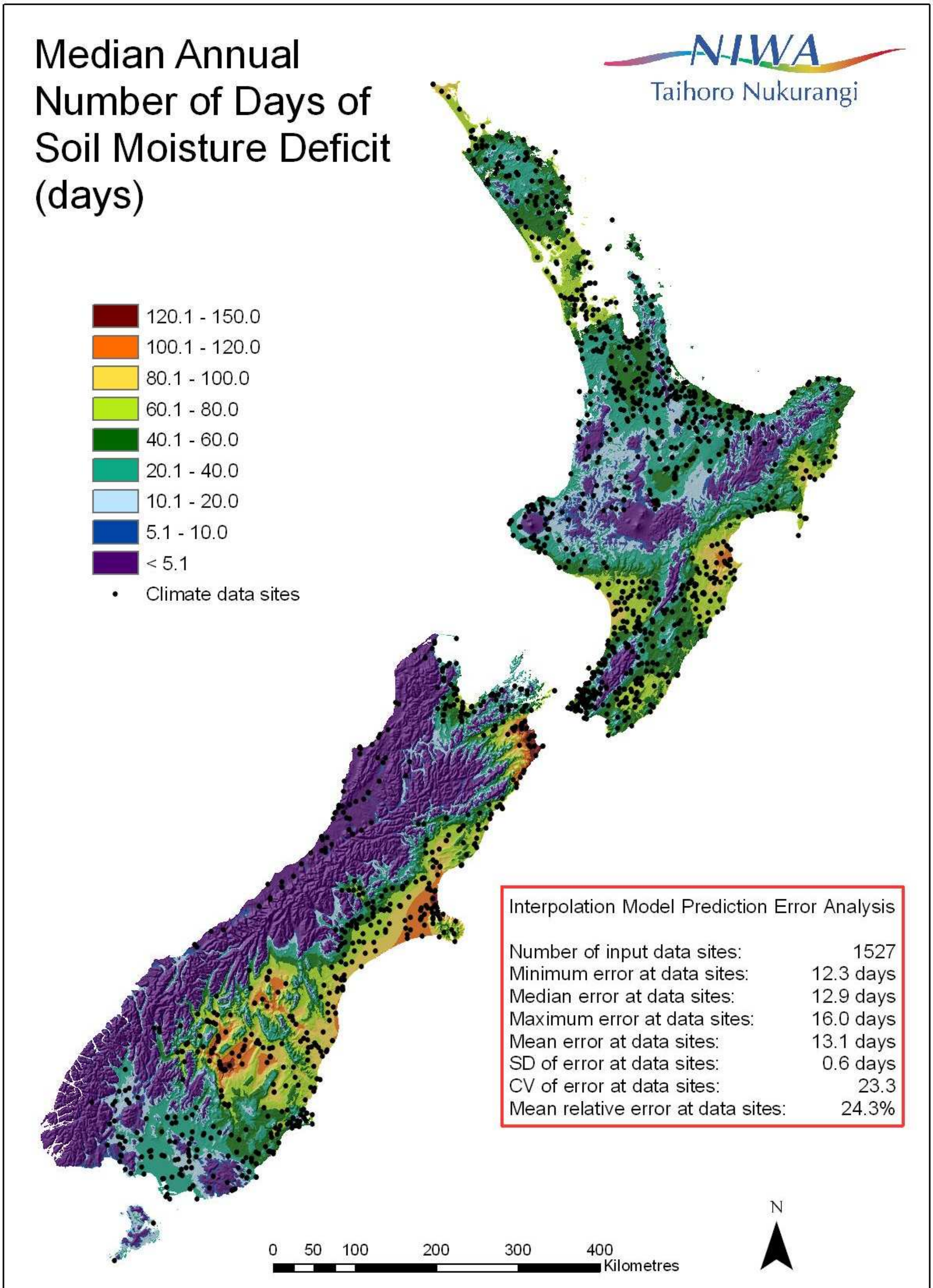


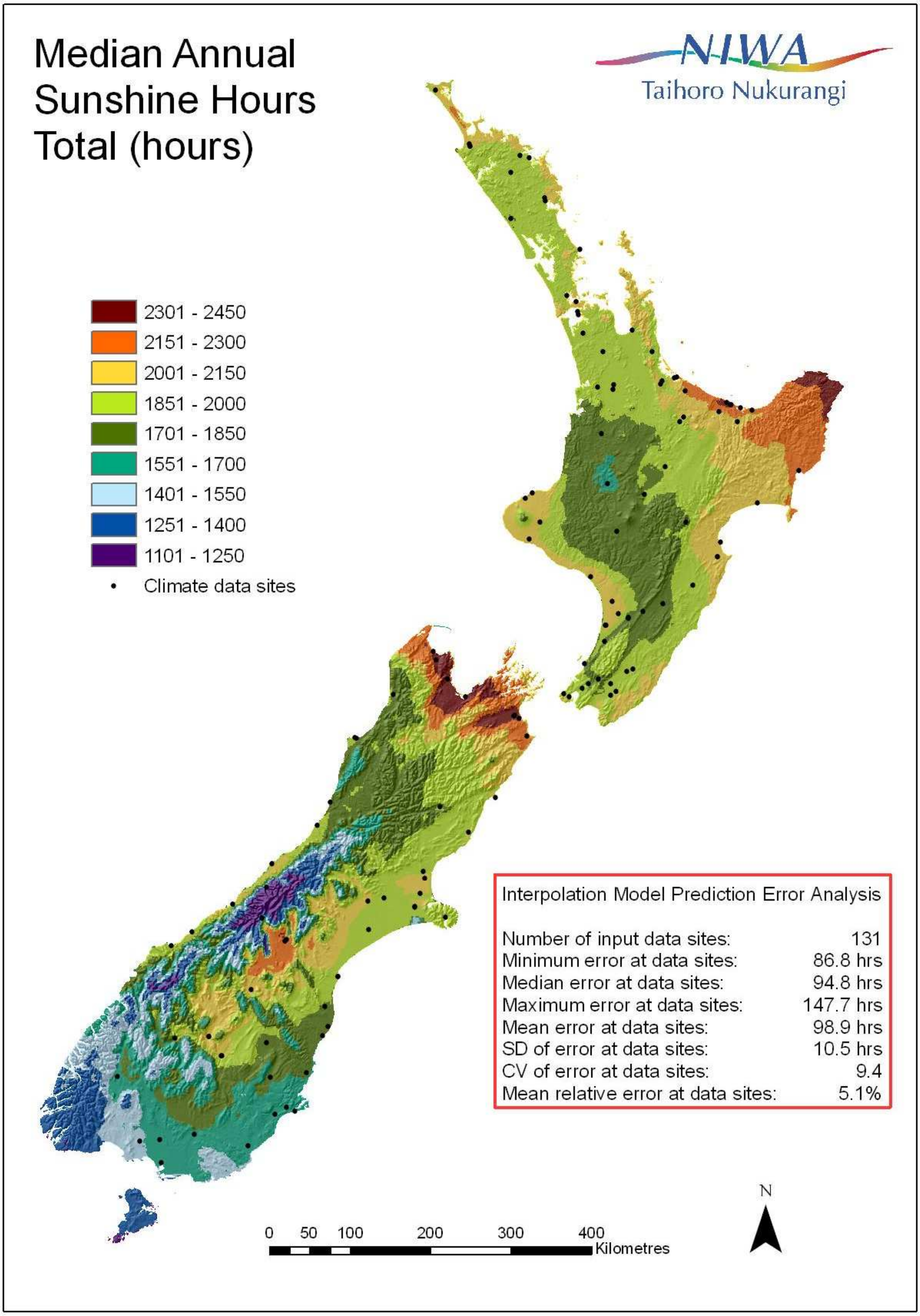
Median Annual
PET Total (mm)
NIWA
Taihoro Nukurangi
901 - 950
851 - 900
801 - 850
751 - 800
701 - 750
651 - 700
601 - 650
551 - 600
501 - 550
401 - 500
< 401
Climate data sites
Interpolation Model Prediction Error Analysis
Number of input data sites: 179
Minimum error at data sites: 27.3 mm
Median error at data sites: 28.8 mm
Maximum error at data sites: 38.8 mm
Mean error at data sites: 29.4 mm
SD of error at data sites: 2.0 mm
CV of error at data sites: 14.6
Mean relative error at data sites: 4.1%
0 50 100 200 300 400
Kilometres
N

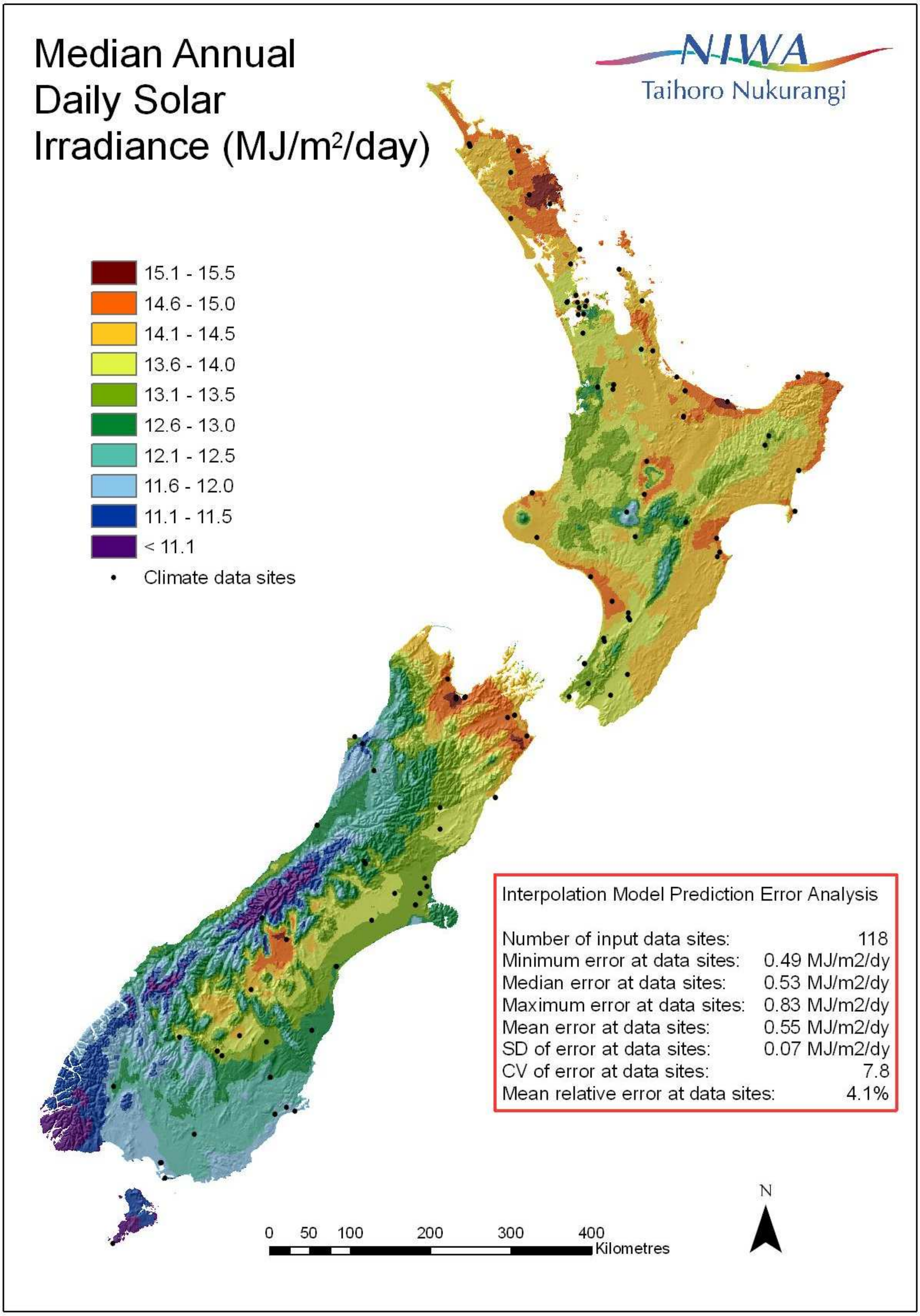

Median Annual
Relative Humidity
at 9am (%)
NIWA
Taihoro Nukurangi
90.1 - 94.0
88.1 - 90.0
86.1 - 88.0
84.1 - 86.0
82.1 - 84.0
80.1 - 82.0
78.1 - 80.0
76.1 - 78.0
74.1 - 76.0
70.1 - 74.0
Climate data sites
Interpolation Model Prediction Error Analysis
Number of input data sites: 515
Minimum error at data sites: 2.3%
Median error at data sites: 2.5%
Maximum error at data sites: 3.5%
Mean error at data sites: 2.6%
SD of error at data sites: 0.2%
CV of error at data sites: 12.1
Mean relative error at data sites: 3.2%
0 50 100 200 300 400
Kilometres
N

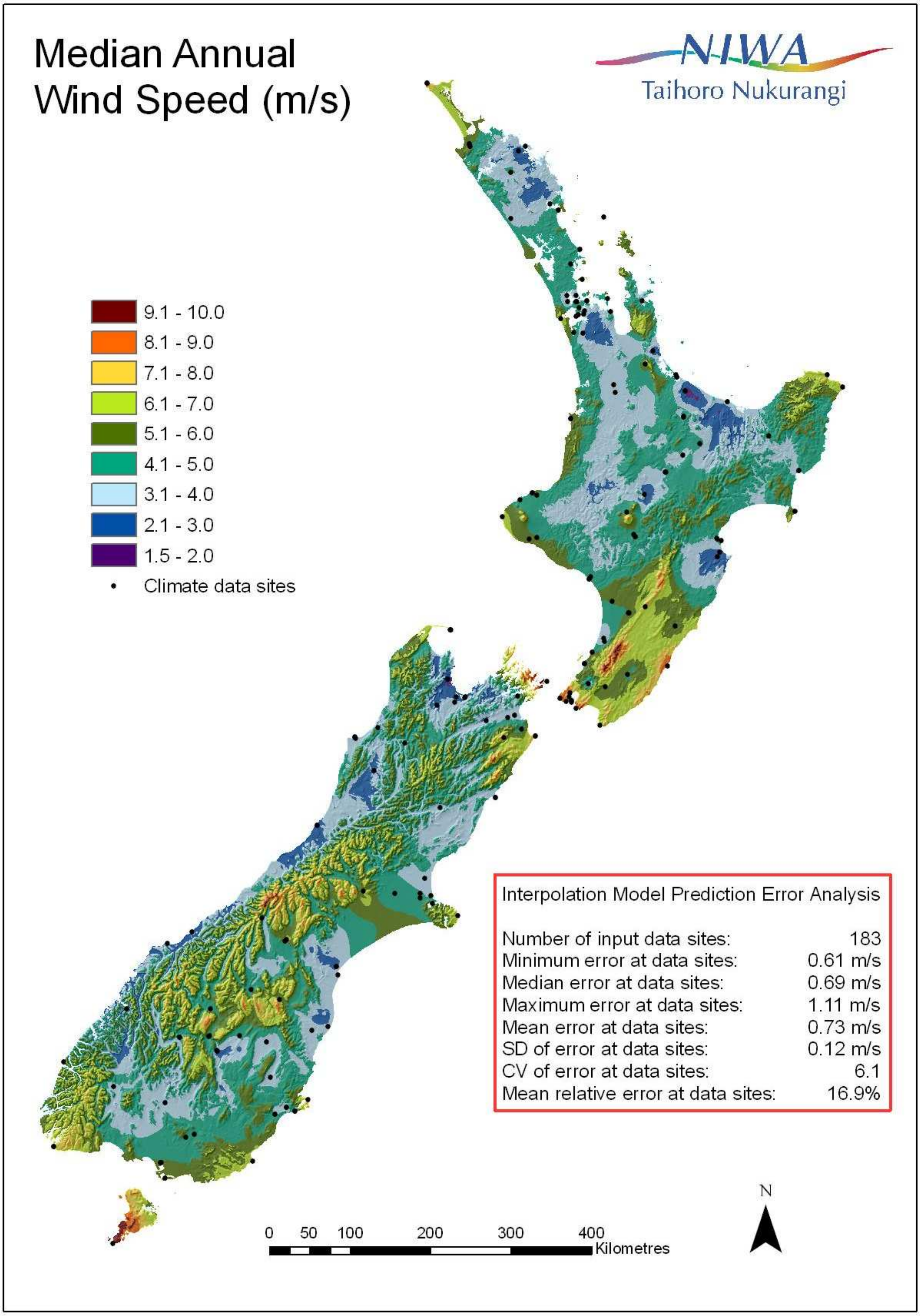

Median Annual
Rainfall Total (mm)
NIWA
Taihoro Nukurangi
6001 - 12500
4001 - 6000
3001 - 4000
2001 - 3000
1501 - 2000
1251 - 1500
1001 - 1250
751 - 1000
501 - 750
301 - 500
Climate data sites
Interpolation Model Prediction Error Analysis
Number of input data sites: 1950
Minimum error at data sites: 132.6 mm
Median error at data sites: 143.0 mm
Maximum error at data sites: 226.5 mm
Mean error at data sites: 147.9 mm
SD of error at data sites: 13.8 mm
CV of error at data sites: 10.7
Mean relative error at data sites: 11.3%
0 50 100 200 300 400
Kilometres
N


Median Annual
Number of Days
or Rain ≥ 1mm
(days)
NIWA
Taihoro Nukurangi
241 - 280
201 - 240
181 - 200
161 - 180
141 - 160
121 - 140
101 - 120
81 - 100
61 - 80
Climate data sites
Interpolation Model Prediction Error Analysis
Number of input data sites: 1621
Minimum error at data sites: 10.1 days
Median error at data sites: 10.8 days
Maximum error at data sites: 14.7 days
Mean error at data sites: 11.1 days
SD of error at data sites: 0.72 days
CV of error at data sites: 15.3
Mean relative error at data sites: 9.3%
0 50 100 200 300 400
Kilometres
N


Median Annual Number of Days of Soil Moisture Deficit (days)
NIWA
Taihoro Nukurangi
120.1 - 150.0
100.1 - 120.0
80.1 - 100.0
60.1 - 80.0
40.1 - 60.0
20.1 - 40.0
10.1 - 20.0
5.1 - 10.0
< 5.1
Climate data sites
Interpolation Model Prediction Error Analysis
Number of input data sites: 1527
Minimum error at data sites: 12.3 days
Median error at data sites: 12.9 days
Maximum error at data sites: 16.0 days
Mean error at data sites: 13.1 days
SD of error at data sites: 0.6 days
CV of error at data sites: 23.3
Mean relative error at data sites: 24.3%
0 50 100 200 300 400
Kilometres
N


Median Annual
Sunshine Hours
Total (hours)
NIWA
Taihoro Nukurangi
2301 - 2450
2151 - 2300
2001 - 2150
1851 - 2000
1701 - 1850
1551 - 1700
1401 - 1550
1251 - 1400
1101 - 1250
Climate data sites
Interpolation Model Prediction Error Analysis
Number of input data sites: 131
Minimum error at data sites: 86.8 hrs
Median error at data sites: 94.8 hrs
Maximum error at data sites: 147.7 hrs
Mean error at data sites: 98.9 hrs
SD of error at data sites: 10.5 hrs
CV of error at data sites: 9.4
Mean relative error at data sites: 5.1%
0 50 100 200 300 400
Kilometres
N


Median Annual
Daily Solar
Irradiance (MJ/m²/day)
NIWA
Taihoro Nukurangi
15.1 - 15.5
14.6 - 15.0
14.1 - 14.5
13.6 - 14.0
13.1 - 13.5
12.6 - 13.0
12.1 - 12.5
11.6 - 12.0
11.1 - 11.5
< 11.1
Climate data sites
Interpolation Model Prediction Error Analysis
Number of input data sites: 118
Minimum error at data sites: 0.49 MJ/m2/dy
Median error at data sites: 0.53 MJ/m2/dy
Maximum error at data sites: 0.83 MJ/m2/dy
Mean error at data sites: 0.55 MJ/m2/dy
SD of error at data sites: 0.07 MJ/m2/dy
CV of error at data sites: 7.8
Mean relative error at data sites: 4.1%
0 50 100 200 300 400
Kilometres
N


Median Annual
Wind Speed (m/s)
NIWA
Taihoro Nukurangi
9.1 - 10.0
8.1 - 9.0
7.1 - 8.0
6.1 - 7.0
5.1 - 6.0
4.1 - 5.0
3.1 - 4.0
2.1 - 3.0
1.5 - 2.0
Climate data sites
Interpolation Model Prediction Error Analysis
Number of input data sites: 183
Minimum error at data sites: 0.61 m/s
Median error at data sites: 0.69 m/s
Maximum error at data sites: 1.11 m/s
Mean error at data sites: 0.73 m/s
SD of error at data sites: 0.12 m/s
CV of error at data sites: 6.1
Mean relative error at data sites: 16.9%
0 50 100 200 300 400
Kilometres
N

4. References

Coulter, J.D. (1975) The Climate. *Biogeography and Ecology of New Zealand.* G. Kuschel, Ed., Junk B.V. Publishers, 87–138.

Hutchinson, M.F. (1989) A new objective method for spatial interpolation of meteorological variables from irregular networks applied to the estimation of monthly mean solar radiation, temperature, precipitation and windrun. *CSIRO Division Water Resources Tech. Memo*, 89/5, pp 95–104.

Hutchinson, M.F. (1991) The application of thin plate smoothing splines to continent-wide data assimilation. *BMRC Research Report* 27, Bureau of Meteorology, Melbourne, pp104–113.

Hutchinson, M.F. (1995). Interpolating mean rainfall using thin plate smoothing splines. *Int. J. Geographical Information Systems*, 9(4): 385–403.

Hutchinson, M.F. 2007. *ANUSPLIN*, http://cres.anu.edu.au/outputs/anusplin.php. Last accessed May 2007.

Salinger, M.J., Gray, W., Mullan, A.B. and Wratt, D.S. (2004) Atmospheric circulation and precipitation. In Mosley, M.P. (Editor), *Freshwaters of New Zealand.* NZ Hydrological Society and Limnological Society of New Zealand, pp 2.1–2.18.

Tait, A.; R. Henderson; R. Turner and X. Zheng (2006) Thin-plate smoothing spline interpolation of daily rainfall for New Zealand using a climatological rainfall surface. *International Journal of Climatology*, 26, pp 2097–2115.

Wratt, D.S., Tait, A., Griffiths, G., Espie, P., Jessen, M., Keys, J., Ladd, M., Lew, D., Lowther, W., Mitchell, N., Morton, J., Reid, J., Reid, S., Richardson, A., Sansom, J., Shankar, U. (2006) Climate for Crops: Integrating climate data with information about soils and crop requirements to reduce risks in agricultural decision-making. *Meteorological Applications*, 13, 305–315.